Three-alpha structures in ^{12}C

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Abstract

We search for three-alpha resonances in ^{12}C by using the complex scaling method in a microscopic cluster model. All experimentally known low-lying natural-parity states are localized. For the first time we unambiguously show that the 0_2^+ state in ^{12}C, which plays an important role in stellar nucleosynthesis, is a genuine three-alpha resonance.

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I. INTRODUCTION

Carbon, which is the fundamental basis of the chemistry of terrestrial life, is produced in red giant stars by burning the helium ash of hydrogen fusion. In order to produce carbon in stellar nucleosynthesis, the $A = 5$ and $A = 8$ nuclear mass-stability gaps must be bridged. Salpeter and Öpik pointed out [1] that the lifetime of ^{8}Be is long enough, so that the $\alpha + \alpha \rightleftharpoons {}^8$Be reaction can produce macroscopic amounts of equilibrium ^{8}Be in stars. Then, the unstable ^{8}Be can capture an additional α particle to produce stable ^{12}C. However, this so-called triple-alpha reaction has very low rate because of the low density of ^{8}Be.

Hoyle argued [2] that in order to explain the measured abundance of carbon in the Universe, this reaction must proceed through a hypothetical resonance of ^{12}C, thus strongly enhancing the cross section. Hoyle suggested that this resonance is a $J^\pi = 0^+$ state at $E_r = 0.4$ MeV (throughout this paper E_r denotes resonance energy in the center-of-mass frame relative to the three-alpha threshold, while Γ denotes the full width). Subsequent experiments indeed found a 0^+ resonance in ^{12}C in the predicted energy region. It is the second 0^+ state (0_2^+) and the second excited state of ^{12}C. Its modern parameters $E_r = 0.3796$ MeV and $\Gamma = 8.5 \times 10^{-6}$ MeV [3] agree well with the old theoretical prediction.

We mention here that the long lifetime of the ^{8}Be ground state and the existence of the 0_2^+ resonance in ^{12}C at the right energy region are only parts of an incredible chain of fortunate nuclear coincidences, which makes the abundant existence of carbon and oxygen possible. For an interesting account, see Ref. [4].

The aim of the present work is to explore the nature of the 0_2^+ state in ^{12}C. The facts that this state is very close to the three-alpha threshold, and that the alpha particle is strongly bound make it probable that the wave function of 0_2^+ has a dominant three-alpha clustering nature. The low-lying states of ^{12}C, including 0_2^+, have been studied in a number of macroscopic (with structureless alpha particles) [5,7] and microscopic [8] three-alpha models. These models reproduce the general features of the low-lying ^{12}C spectrum. However, all these models without exception assume three-body bound state- or two-body ^{8}Be $+ \alpha$ scattering state asymptotics for the wave functions. Thus, none of them obeys the physically correct three-body boundary condition for states above the three-alpha threshold. Those models which use ^{8}Be $+ \alpha$ asymptotics with bound-state-like ^{8}Be [8] are seemingly adequate, because the small width of the ^{8}Be ground state makes its wave function very similar to a bound state wave function in a large spatial region. However, one must realize that such a model predicts the states above the three-alpha threshold, e.g. 0_2^+, to be two-body, ^{8}Be $+ \alpha$, resonances. This means that currently there is no unambiguous evidence that these resonances are intrinsic states of ^{12}C. In fact, it was speculated that the 0_2^+ state is not a three-body resonance, but an enhancement coming from the ^{12}C $\rightarrow {}^8$Be $+ \alpha \rightarrow \alpha + \alpha + \alpha$ sequential decay [9]. This idea was, however, criticized [10] by arguing that all the experimental data supported the genuine ^{12}C nature of this state.

In the present paper we use a method which is able to handle the three-body dynamics of the 0_2^+ state correctly. Thus for the first time we can unambiguously show whether this state is a genuine three-alpha resonance in ^{12}C. We also study other low-lying natural-parity states of ^{12}C.

II. MODEL

Our model is a microscopic three-cluster ($\alpha + \alpha + \alpha$) resonating group method (RGM) approach to the twelve-nucleon system. The trial function of the twelve-body problem has the form

$$\Psi^{^{12}\text{C}} = \sum_{l_1,l_2} \mathcal{A}\left\{\Phi^\alpha \Phi^\alpha \Phi^\alpha \chi^{\alpha(\alpha\alpha)}_{[l_1 l_2]L}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2)\right\}, \tag{1}$$

where $\mathcal{A}$ is the intercluster antisymmetrizer, the Φ^α cluster internal states are translationally invariant $0s$ harmonic-oscillator shell-model states with zero total spin, the $\boldsymbol{\rho}$ vectors are the intercluster Jacobi coordinates, l_1 and l_2 are the angular momenta of the two relative motions, L is the total orbital angular momentum and $[\ldots]$ denotes angular momentum coupling. The total spin and parity of ^{12}C are $J = L$ and $\pi = (-1)^{l_1 + l_2}$, respectively.

Putting (1) into the twelve-nucleon Schrödinger equation which contains the nucleon-nucleon ($N - N$) strong and Coulomb interactions, we get an equation for the intercluster relative motion functions χ. These functions represent the three-body dynamics of the ^{12}C states. In order to determine these functions, we have to use a method which can handle three-body resonances.

There are indirect and direct approaches. The indirect methods, e.g. [11], study the three-body problem at real energies, and extract resonance parameters from the three-body phase shifts. A similar method has recently been used to study the 0_2^+ state of ^{12}C [7]. However, the resonant nature of this state and the resonance parameters were extracted from a bound state wave function by using the WKB approximation.

The aim of the direct methods for resonances is to find the complex-energy poles of the three-body scattering matrix. For example, in [12–14] the authors determined the pole positions of the three-body S-matrix by analytically continuing the homogeneous Faddeev-equation to complex energies. To get a decisive answer to the question of the nature of the 0_2^+ state, we must use a direct method.

Our choice is the complex scaling method (CSM) [15]. It reduces the problem of asymptotically divergent resonant states to that of bound states, and can handle the Coulomb interaction without any problem. The main point of the CSM is, that instead of solving the original Schrödinger equation for resonances, a new Hamiltonian is defined by

$$\widehat{H}_\theta = \widehat{U}(\theta)\widehat{H}\widehat{U}^{-1}(\theta), \tag{2}$$

and the complex equation

$$\widehat{H}_\theta|\Psi_\theta\rangle = \varepsilon|\Psi_\theta\rangle \tag{3}$$

is solved. In coordinate space the unbounded similarity transformation $\widehat{U}(\theta)$ acts on a function $f(r, \hat{r})$ as

$$\widehat{U}(\theta)f(r, \hat{r}) = e^{3i\theta/2}f(re^{i\theta}, \hat{r}), \tag{4}$$

where $\hat{r}$ describes the angular part of $\mathbf{r}$. For real angles θ, $\widehat{U}(\theta)$ results in a rotation into the complex coordinate plane, whereas for complex θ, $\widehat{U}(\theta)$ results in a rotation and scaling.

Further on, we shall always use real θ values. In the case of a many-body problem, the transformation given by Eq. (4) has to be performed in each dynamical Jacobi coordinate.

For a broad class of potentials there is a remarkable connection between the spectra of $\widehat{H}$ and $\widehat{H}_\theta$ [16]: (i) the bound eigenstates of $\widehat{H}$ are eigenstates of $\widehat{H}_\theta$, for any value of θ within $0 \leq \theta \leq \pi/2$; (ii) the continuous spectrum of $\widehat{H}$ is rotated by an angle 2θ; (iii) the complex generalized eigenvalues of $\widehat{H}_\theta$, $\varepsilon_{\mathrm{res}} = E_{\mathrm{r}} - i\Gamma/2$ (with E_{r}, $\Gamma > 0$), belong to its proper spectrum, with square-integrable eigenfunctions, provided $2\theta > |\arg \varepsilon_{\mathrm{res}}|$. These complex eigenvalues coincide with the S-matrix pole positions.

In nuclear physics the CSM has been successfully applied to two-body problems, like in a RGM description of ^{8}Be [17], in an OCM model of ^{20}Ne [18] and in the OCM description of the resonances of ^{10}Li [19]. It was also tested for three-body resonances [20], and was used, e.g., in a RGM model of ^{6}He, ^{6}Li and ^{6}Be [21], in searching for three-nucleon resonances [22], in three-body models of ^{6}He, ^{10}He and ^{11}Li [23] and in an RGM model of ^{9}Be and ^{9}B [24]. Further details and references of the method can be found there. We note, that the CSM is identical to a contour rotation in momentum space [25]. The latter method was also used, for example, to study three-body resonances in the A = 6 nuclei [13].

Up to Eq. (3) our treatment of three-body resonances is exact. Since the resonant wave functions become square-integrable in the CSM, we can use any bound-state method to describe them. We expand the relative motion functions χ in Eq. (1) in terms of products of tempered Gaussian functions, $\rho_1^{l_1} \exp[-(\rho_1/\gamma_i)^2] Y_{l_1 m_1}(\widehat{\rho}_1) \cdot \rho_2^{l_2} \exp[-(\rho_2/\gamma_j)^2] Y_{l_2 m_2}(\widehat{\rho}_2)$ (where l_1 and l_2 are the angular momenta in the two relative motions, respectively, and the widths γ of the Gaussians are the parameters of the expansion), and determine the expansion coefficients from the $\langle \delta\Psi_\theta | \widehat{H}_\theta - \varepsilon | \Psi_\theta \rangle = 0$ projection equation. This way we discretize the continuum and select the square-integrable solutions of Eq. (3). We use ten Gaussian basis functions in each relative motion. The matrix elements of the complex scaled many-body Hamiltonians were calculated in exact analytic forms by using computer algebraic techniques.

III. RESULTS

In order to avoid any possible model dependence of the conclusions we use three different effective $N - N$ interactions. The Minnesota (MN) force was designed to reproduce low-energy $N - N$ scattering data [26], while the rather different Volkov 1 (V1) and 2 (V2) forces were obtained from fitting the bulk properties of s- and p-shell nuclei [27]. Each force contains an exchange mixture parameter, u and m, respectively. We fix these parameters by requiring that the energy of the ^{8}Be ground state be reproduced. The harmonic-oscillator size parameters of the alpha particle internal states are chosen to minimize the free-alpha energies. Thus, the wave function of the alpha particle is variationally stabilized. The size parameters of the alpha particle and the exchange mixture parameters of the $N - N$ forces are listed in Table 1 for the three interactions, together with the energies and radii of the alpha particle.

The parameters of the low-lying ^{8}Be resonances, given by the MN, V1 and V2 forces, are shown in Table 2. For ^{8}Be we use a two-alpha cluster model wave function, similar to Eq. (1)

$$\Psi^{^8\mathrm{Be}} = \mathcal{A}\Big\{\Phi^\alpha \Phi^\alpha \chi_L^{\alpha\alpha}(\boldsymbol{\rho})\Big\}. \tag{5}$$

The relative motion function χ is determined by using a two-body scattering approach based on the Kohn-Hulthén variational method [28]. The $\alpha + \alpha$ scattering phase shifts, coming from the MN interaction, are shown in Fig. 1, together with the experimental data. A nice agreement is observed. The resulting scattering matrices are continued to complex energies, where their poles are localized [29]. The resonance parameters in Table 2 were extracted from the complex pole positions.

This analytic continuation method is in principle equivalent with the CSM, but is numerically far more precise. The CSM has difficulties in localizing very narrow resonances. As an illustrative example, we performed CSM calculations for the ^{8}Be resonances and found that although the energy of the ground state is well reproduced, the width is strongly overestimated. For θ values which give stable complex energy spectra, with the discretized continuum rotated by close to 2θ, this overestimation is more than two orders of magnitude. We could fine-tune the θ angle and the Gaussian basis parameters to get the experimental widths, but then the discretized continuum points would be scattered, forming a band rather than a line. If we used such a θ value in the ^{12}C calculations, the results would be disastrous, making the identification of the resonances impossible. Therefore, we use such θ values in the ^{12}C calculations which give more or less stable discretized continua. The price we have to pay for this is that we cannot resolve very small widths. However, this is not a serious problem, because our primary goal is to show the *existence* of the ^{12}C states, and not to determine the precise resonance parameters. Our model and effective interactions are probably inadequate for this latter purpose.

The $N-N$ interactions, thus set to reproduce the unbound ^{8}Be ground state, are used in the ^{12}C calculations. In this way we ensure that there is no bound two-body subsystem in the three-alpha system as shown also by the experimental data. Therefore, our model handles the three-body dynamics properly, and the resonances we find are genuine three-alpha structures.

We perform calculations for the low-lying natural-parity states of ^{12}C. In the three-alpha wave function (1) we include $l_1 = 0$ for the 0^+ and 2^+ states, and additionally $l_1 = 2$ for the 1^- and 3^- states. Our test calculations show that the addition of further configurations hardly influences the results. For instance, if we include the $[l_1, l_2]L = [2,2]0$ configuration in the 0_1^+ ground state wave function in addition to the $[0,0]0$ one, we gain less than 1% in the three-body binding energy and less than 0.1% in the absolute energy of ^{12}C. This is in sharp contrast to the findings of macroscopic models [5] which assume structureless alpha particles. We believe that the antisymmetrization causes the $[0,0]0$ state to strongly dominate.

The parameters of the three-body resonances we find, using the MN, V1 and V2 forces are listed in Table 3, together with the experimental values. The ground state of ^{12}C is strongly overbound by MN, roughly reproduced by V1 and underbound by V2. Interactions V1 and V2 give almost the same *absolute* energy for the ground state, so the difference between the energies relative to the three-alpha threshold comes from the different alpha particle energies shown in Table 1. However, this is not the case for the differences between the MN and V1 and V2 ground state energies. The reason of the differences between the MN and Volkov results is most probably the different exchange mixture structure of these forces.

While the MN force reproduces the deuteron binding energy in an $L = 0$ space, assumed here, the Volkov forces underbind the deuteron. However, the Volkov forces unphysically bind the singlet dinucleon states.

We believe that the ^{12}C ground state should be expected to be overbound in three-alpha models. The reason is that the ground state of ^{8}Be is not a perfect two-alpha state. It has been shown that the inclusion of ^{7}Li $+ p$ and ^{7}Be $+ n$ channels in the ground state wave function of ^{8}Be significantly increases its "binding energy" [30]. Thus, in order to reproduce the ^{8}Be ground state energy in a model which contains these configurations, the $N - N$ interaction (or the $\alpha - \alpha$ interaction in macroscopic models) should be weakened. We expect that then the 0_1^+ state of ^{12}C would be closer to the experimental position, using the MN force, even if the ^{12}C wave function also contains higher-lying rearrangement channels. We note, that all macroscopic models underbind the 0_1^+ state of ^{12}C [5]. The only exception is Ref. [6] where the "microscopic" $\alpha - \alpha$ potential was constructed in a way which took into account some of the effects of the internal structure of the alpha particle. In agreement with our MN result, Ref. [6] found the ground state of ^{12}C to be overbound.

Table 3 shows that all experimentally known low-lying natural-parity states of ^{12}C are reproduced by our model. This means that these states are all genuine three-alpha resonances. In many cases the calculated resonance parameters are far from the experimental values. In order to get closer to the experiments, major improvements of the model, e.g., the inclusion of rearrangement channels with nonzero spin, would be necessary.

Our most important result is that we can localize the 0_2^+ state as a three-body resonance. In Fig. 2 we show the low-energy part of the 0^+ spectrum of the complex scaled Hamiltonian with the MN interaction. The two three-body resonances are denoted by the circles. The width of the 0_2^+ state is probably overestimated by the CSM (cf. the discussion about the CSM and very small widths). The dots in Fig. 2 represent the rotated discretized three-body cut. Due to numerical precision problems these points form a band rather than a line. We attempted to optimize the Gaussian basis, but could not get a result cleaner than the one shown in Fig. 2. The numerical stability of the continuum points is rather sensitive to the level of precision maintained during the calculations of the complex many-body matrix elements. Although every matrix element was calculated from (very involved) algebraic expressions, no special attention was paid to the full optimization of the computational algorithm against the loss of numerical precision. Nevertheless, the identification of the resonances is unambiguous.

We found that the parameters of the resonances are approximately independent of the rotation angle θ within a reasonable interval. We performed most of the calculations in Table 3 with $\theta = 0.2$ rad, and checked several of them with $\theta = 0.1$ rad. Figure 2 shows the result of a calculation with $\theta = 0.1$ rad. We have encountered some problems regarding the 3^- states. Using $\theta = 0.2$ rad, one of these resonances can be lost. That is why we used $\theta = 0.1$ rad for this state in Table 3.

In contrast to the ground state of ^{12}C, the 0_2^+ state is predicted by all three interactions at roughly the same position, and close to the experiment. This is not surprising, because the 0_2^+ state, being so close to the three-alpha threshold, is expected to be a more perfect three-alpha state than the ground state.

We would like to note, that although the 0_2^+ state is a three-body resonance, it does not decay into an uncorrelated three-alpha final state. Experiments show that the decay

proceeds predominantly through the $^8\text{Be} + \alpha \rightarrow \alpha + \alpha + \alpha$ sequential process [31]. The experimental upper limit for the contribution of the three-alpha decay to the alpha decay width of the 0_2^+ state is less then 4%. The dominance of the $^8\text{Be} + \alpha$ decay over the 3α one is the result of the difference between the relative phase spaces [31].

IV. CONCLUSION

In summary, we have studied the resonances of ^{12}C in a microscopic three-alpha model. We used the complex scaling method, which allowed us to describe the three-body Coulomb dynamics correctly for resonances. We used three different effective nucleon-nucleon interactions and their results are consistent with each other. We have localized all experimentally known low-lying natural-parity states in ^{12}C, although a better agreement with experiment would require major improvements of our model. For the first time we were able to unambiguously show that the 0_2^+ state of ^{12}C, which plays an important role in the astrophysical triple-alpha process, is a genuine three-alpha resonance.

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FIGURES

FIG. 1. Calculated $\alpha + \alpha$ scattering phase shifts in the center-of-mass frame using the MN interaction. Experimental data are taken from Ref. [3].

FIG. 2. Low-energy eigenvalues of the complex scaled Hamiltonian of the 0^+ three-alpha states in ^{12}C. The dots are the points of the rotated discretized continuum, while the circles are three-alpha resonances. The rotation angle is 0.1 rad.

TABLES

TABLE I. Harmonic-oscillator size parameter β of the α internal state, exchange mixture parameter u or m of the $N-N$ interaction, and the energy E_α and point nucleon rms radius of the free α particle.

	β (fm^{-2})	u or m	E_α (MeV)	r_α (fm)
MN	0.6060	0.93344	−24.687	1.36
V1	0.5291	0.57286	−27.085	1.46
V2	0.5284	0.60126	−27.957	1.46
Experiment	—	—	−28.269	1.48

TABLE II. Energies (relative to the two-alpha threshold) and full widths of low-lying resonances in ^{8}Be. All quantities are given in MeV.

	MN		V1		V2		Experiment [3]	
	E	Γ	E	Γ	E	Γ	E	Γ
0^+	0.092	6.15×10^{-6}	0.092	2.36×10^{-6}	0.092	5.17×10^{-6}	0.09189	$(6.8\pm1.7)\times10^{-6}$
2^+	3.03	1.39	2.34	1.48	2.26	1.42	3.04±0.03	1.50±0.02
4^+	13.10	4.11	9.96	5.89	9.55	5.93	11.4±0.03	∼3.5

TABLE III. Energies (relative to the three-alpha threshold) and full widths of low-lying natural-parity three-body resonances in ^{12}C. All quantities are given in MeV.

	MN		V1		V2		Experiment [3]	
	E	Γ	E	Γ	E	Γ	E	Γ
0^+	−10.43[a]		−7.56[a]		−5.27[a]		−7.2746[a]	
	0.64	0.014	0.71	0.031	0.83	0.077	0.3796±0.0002	$(8.5\pm1.0)\times10^{-6}$
	5.43	0.92	4.75	0.75	4.68	0.89	3.0±0.3	3.0±0.7
	16.01	1.74	15.44	2.89	15.91	3.71	10.49±0.02	0.08±0.02
2^+	−7.63[a]		−5.13[a]		−2.47[a]		−2.8357±0.0003[a]	
	6.39	1.10	5.55	1.11	5.49	1.54	3.89±0.05	0.43±0.08
3^-	1.16	0.025	1.35	0.003	1.85	0.014	2.366±0.005	0.034±0.005
	11.91	1.69	12.18	1.41	13.80	2.79	11.08±0.05	0.22±0.05
1^-	3.71	0.36	3.72	0.47	3.82	0.72	3.569±0.016	0.315±0.025

[a]Bound state.